A Comparison of Regularized Metrics for Phase Diversity

Matthew R. Bolcar and James R. Fienup
The Institute of Optics, University of Rochester, Rochester, NY 14627
mrbolcar@gmail.com

Abstract: We compare the performance of four metrics for use in a phase diversity algorithm. Three of the metrics utilize a regularization based on the signal-to-noise ratio.

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1. Introduction

Phase diversity algorithms estimate system aberrations and an unknown object from multiple, diverse blurred images [1,2]. In order to reduce the amount of computation during the optimization process of a phase diversity algorithm, the unknown object parameters can be explicitly removed from the metric by the reduced Gaussian (RG) technique, resulting in a metric that depends only on the unknown phase parameters and is given by

$$E(\boldsymbol{\alpha}) = \sum_{u,v}\sum_{k=1}^{K}\left|D_k(u,v)\right|^2 - \sum_{u,v\in\chi}\left[\left|\sum_{j=1}^{K} D_j(u,v)\,S_j^*(u,v;\boldsymbol{\alpha})\right|^2 \Big/ \sum_{m=1}^{K}\left|S_m(u,v;\boldsymbol{\alpha})\right|^2\right] \quad (1)$$

where $\boldsymbol{\alpha}$ is a vector of the unknown phase parameters, (u,v) are spatial frequency coordinates, K is the number of diversity images, D_k is the Fourier transform of the k^{th} detected image, S_k is the k^{th} optical transfer function (OTF), and χ is the set of spatial frequencies at which the denominator does not equal zero.

The limit of the second summation in Eq. (1) over χ acts as a regularization term. However, the metric may still become unstable in regions of low signal-to-noise ratio (SNR). To mitigate this, several metrics have been developed that utilize regularizations based on the object and noise power spectra. In this paper we compare four such metrics, including the metric of Eq. (1), with respect to computation time and error in the phase estimation.

2. Regularization Methods

Two of the metrics, developed by Blanc et al. [3], are derived from maximum *a posteriori* likelihood functions that include statistical models for the object and phase. In the first, called the joint maximum *a posteriori* (JMAP) metric, the reduced Gaussian technique is applied to the joint likelihood function. In the second, called the marginal *a posteriori* (mAP) metric, the object is integrated out of the joint likelihood function. In both cases, the resulting metric depends only on the unknown phase parameters but retains information on the SNR in the form of the object and noise power spectra.

A third metric, a regularized reduced Gaussian (RRG) metric, is derived from the Gaussian maximum likelihood function. This is similar to the derivation of Eq. (1), except a Wiener-Helstrom filtered estimate of the object incorporating the object and noise power spectra is used in the reduced Gaussian method, instead of the traditional inverse filtered estimate.

3. Results and Conclusions

The table below summarizes the results of each of the four metrics described above. Two diversity images were simulated with the average pixel in each image having an SNR of ~20. Note the MAP metric yields the best phase estimate, yet the RG metric provides a very good phase estimate in nearly a third of the time.

Metric:	RG	RRG	JMAP	mAP
RMS Phase Est. Error (waves):	0.088	0.057	0.083	0.045
Relative Computation Time:	1.000	0.307	0.599	0.882

4. Acknowledgements and References

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[1] R.A. Gonsalves and R. Chidlaw, "Wavefront sensing by phase retrieval," in *App. of Digital Image Processing III*, Proc. SPIE **207** (1979).
[2] R.G. Paxman, T.J. Schulz and J.R. Fienup, "Joint estimation of object and aberrations by using phase diversity," *J. Opt. Soc. Am. A* **9**, 1072 – 1085 (1992).
[3] A. Blanc, L. Mugnier and J. Idier, "Marginal estimation of aberrations and image restoration by use of phase diversity," *J. Opt. Soc. Am. A* **20**, 1035 – 1045 (2003).