Comparison of Estimation Methods for Field-dependent Phase Aberrations

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Abstract: Two techniques of multi-field phase retrieval are compared using digital simulations and Fisher Information theoretical methods. Results show solving for phase coefficients with explicit field-dependence yields a lower estimation error in the phase retrieval process.

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1. Introduction

It is not uncommon for multiple scientific instruments to share the focal plane of an optical system, particularly in the case of large, astronomical telescopes. In the case of the James Webb Space Telescope, regular phasing of the telescope will be achieved by performing phase retrieval on images collected by a single instrument near the optical axis. This procedure can result in the system being well corrected only for the FOV covered by that collecting instrument. Meanwhile, instruments near the edge of the FOV may be unacceptably aberrated. Multi-field phase retrieval has been proposed as a solution to this problem [1]. In this paper, we compare two methods of multi-field phase retrieval using digital simulations and an information theoretic approach.

2. Two Methods

Both of the methods being compared in this paper involve a parametric approach to phase retrieval, in which nonlinear optimization is used to estimate the coefficients of a polynomial expansion of the unknown phase errors in the system. The first method involves expressing the unknown wavefront as

$$W\left(r_f,\theta_f;\rho,\phi\right)=\sum_n \beta_n\left(r_f,\theta_f\right)Z_n\left(\rho,\phi\right) \tag{1}$$

where (r_f, θ_f) are the coordinates of the f^{th} field point, (ρ, ϕ) are the pupil coordinates, β are the unknown, field-dependent phase coefficients, and Z_n is the n^{th} phase basis function (e.g. Zernike polynomial). For a single point source at field coordinate (r_F, θ_F), an image would be detected and the unknown phase coefficients $\beta(r_F, \theta_F)$ would be estimated. Repeating this process for several point sources at different field points, one could eventually determine the field-dependent aberrations of the system and correct the system over the entire FOV.

A second method of representing the wavefront is to explicitly include the dependence of the aberration on field in the polynomial expansion. For example, Kwee & Braat [2] demonstrate the form

$$W\left(r,\theta;\rho,\phi\right)=\sum_m\sum_n \alpha_{m,n}Z_m\left(r,\theta\right)Z_n\left(\rho,\phi\right) \tag{2}$$

where a second polynomial expansion in the field coordinates is included in the term $Z_m(r, \theta)$. Now a single set of coefficients, α, will represent the wavefront, regardless of the field coordinate, unlike Eq. (1) where a different set of coefficients is required for each individual field point. The β coefficients in Eq. (1) can be linearly related to the α coefficients, which is necessary for comparing the two methods.

3. Results and Conclusions

The two proposed methods were first compared from an information theoretic perspective. Cramer-Rao lower bounds were calculated for the error in estimating the coefficients using each approach. This theory predicts about 37% lower error in estimating the coefficients with explicit field dependence given in Eq. (2). Digital simulation supports this prediction, showing a 30% lower error in estimating the coefficients α.

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[1] B.H. Dean, D.L. Aronstein, et al. "Multifield wavefront sensing and control for the James Webb Space Telescope" in Proc. SPIE **6265**, Orlando, FL, May 2006.
[2] I.W. Kwee and J.J.M. Braat, "Double Zernike expansion of the optical aberration function," Pure Appl. Opt. **2**, 21 – 32 (1993).